The maths behind the Great Barrier Reef

Author: F. P. Meijvis
Profile: NTG
Subject: Mathematics
School: Het Schoonhovens College
Level: t-VWO
Supervisor: Mrs. Molenaar
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Summary

Climate change has had a destructive effect on the Great Barrier Reef, which is now in need of help. Studies show that the coral cover has declined by about 50% of it's initial cover over the last 27 years[7]. The main causes for decline are: Tropical storms, Bleaching and Crown-Of-Thorns Starfish.

In this paper I have searched for possible solutions to counteract this decline by building a mathematical model of the reef. My model does provide a general solution which we can undertake to help the coral recover, but the model does not provide credible numbers. All this is also assuming most of the made assumptions are correct, which I do not have evidence for.

The model suggested that reducing and controlling the population size of the Crown-Of-Thorns starfish has a positive effect on the coral cover. But this effect will only be present as long as the influence of coral bleaching is limited, which is why we must also undertake action to prevent a rise in global and sea temperatures.

Introduction

In February 2019 I participated in the Junior TU Delft course: Mathematical modeling. This was essentially a crash course in making a mathematical model intended for students who are good at maths and wanted to challenge themselves. This was also one of the reasons why I decided to take part.

The 5 fridays that I was in Delft were some of the most fun school days I've had in the 6 years that I have been in secondary school. So when I had to find a subject for my School Research Project, I knew it had to be something involving a model.

In the media it is often mentioned that climate change is having its effect on earth. One of these consequences is the danger that now threatens the Great Barrier Reef in Australia. The reef is not doing well (that is lightly expressed), but I would still like to visit it in the future. This is why I decided to make a model about the reef for my SRP: To show what I learned in Delft and to figure out whether I would still be able to visit the reef in the future.

This is why my main research question is: *"What will the Great Barrier Reef look like in the future, and what should we do to save it?"* To answer this question, I will first explain what mathematical modeling is. Then I will tell you something about the Great Barrier Reef. And finally, we can make a model about the reef, which we will interpret afterwards to answer our main question. I also consider the making of the model my practical component.

The idea behind this is that we might be able to find a solution to prevent further decay of the coral in the Great Barrier Reef. This is also why it is not of utmost importance that the model is a perfect description of the reef. I want to find a general plan for saving the reef, and to do that it is not necessary to have all the correct numbers.

Table of contents

A mathematical model

The main purpose of a mathematical model is to be able to create an understanding of a complex situation[21]. They are often used to either predict what something will be like in the future, or to investigate the results of a certain action.

Take for example one of the problems I had received at the TU Delft course. A forest that has a population of mice in it. The municipality governing the forest had decided that a highway would be build right through the middle. After the building of the highway a study showed that one side of the road had a population of 1000 mice, and the other 200. The municipality was facing difficulties related to the high population of mice on one side, and decided to build an eco-bridge so the mice could cross the highway. But before spending more money they wanted to know if this eco-bridge would help lower the population on the most populated side, and raise the population on the other.

The mathematician that was given this problem decided to build a model to solve this situation. Models usually consist of (a set of) equations that describe a phenomenon, like the migration of the mice over the eco-bridge in the previous paragraph. The model would give the mathematician insight into the migration, and with that he would be able to answer the question he had been given.

But how did the mathematician build the model? All mathematical models are constructed in many of the so called “modeling cycles”. These cycles are what a mathematician undergoes multiple times before he reaches his final model. One cycle consists of the following four phases: a problem phase, a modeling phase, a calculations phase and a verification / reflection phase. See figure 1.

The problem phase is where you define what you want to make a model about. In the example of the eco-bridge and the mice this would be the migration of the mice, and whether the bridge would help even the populations. Then in the modeling phase he builds the model based on the information that he has. With this model he could calculate what the population of mice in the 2 areas would look like after a certain amount of time: the calculation phase. Then in the final phase he answers the question and checks if his model is realistic and correct.

Figure 1[21]: The modeling cycle

To give you a concrete view, the first modeling cycle of the mice and eco-bridge problem from the Junior TU Delft course has been put below.

Problem phase:

Nature reserves “the high wood” and “the low wood” are being separated by a highway. On a given moment it seems like the high wood has a population of 1000 mice of a protected species and the low wood has but 200 mice. Because the high population in “the high wood” leads to problems, the municipality wants to construct an eco-bridge in the hopes that the mice will spread out more evenly. It appears the migration between both areas is constant. On a monthly basis 5% of the mice go from “the low wood” to “the high wood” and 10% go from “the high wood” to “the low wood”.

The municipality has the following question for you: “what will be the population size after a year?”

Modeling phase:

Using the given information, you can build the following model. With Ph (t) being defined as the population of mice in the “high wood” and Pl (t) being the population in the “low wood” after “t” months.

$$\begin{bmatrix} Ph(t) \\ Pl(t) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.05 \\ 0.1 & 0.95 \end{bmatrix}^t * \begin{bmatrix} 1000 \\ 200 \end{bmatrix}$$

Figure 2, migration of the mice over the bridge on a monthly basis.

Do note: I have not explained how I got to the following model, but in my own model about the Great Barrier Reef I will explain why I take certain actions.

Calculation phase:

The population distribution after 12 months is as follows:

$$\begin{bmatrix} Ph(12) \\ Pl(12) \end{bmatrix} = \begin{bmatrix} 485 \\ 715 \end{bmatrix}$$

Figure 3, mice distribution after 12 months

to acquire this simply fill in t = 12 in the model and let a calculator or computer solve it.

Verification phase:

The model above does accurately represent the migration of the mice, but there are certain things which are not realistic. There are no factors related to growth of the population and death, meaning that the mice in this model are immortal and unable to reproduce. This is in reality obviously not the case , and a new modeling cycle will have to be undergone to solve these problems. These kinds of things are usually discussed in the verification phase.

Many of the models make use of differential equations. These are equations that describe the speed at which something grows[21] (or declines). Models like this are often solved with the help of Euler's method. Euler's method can be described as follows:

1. Take your starting point
2. Solve the differential equation for your starting point. This is the speed at which your point will move.
3. Assume that the speed is constant for a certain period of time. This period is often called Δt (delta t).
4. Calculate where your point would end up after this Δt interval. To do this, simply multiply the solution of the differential equation from step 2 with Δt (you will have calculated the change in the parameter), then add that answer to your starting point.
5. You have now gained a new point, and can return to step 1 to find another.

Since this is a numerical method you do not get a precise answer as you would have gotten when using an algebraïc way. In step 3 you make an assumption which is not correct. The speed changes when the size of the population changes. How smaller your Δt interval is, the more precise your graph will be.

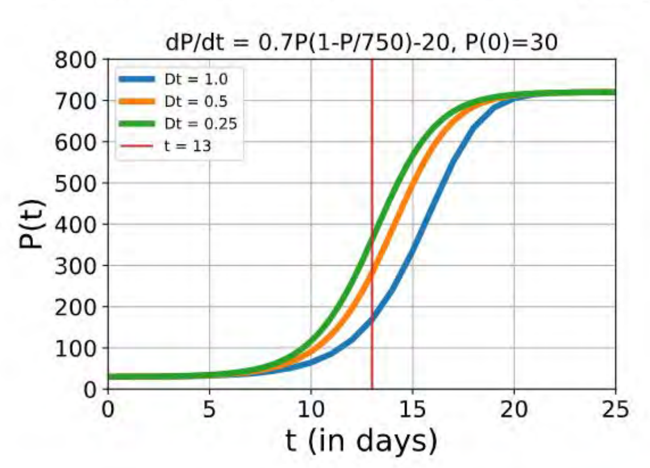

Figure 4: The smaller Δt, the more the numerical solution looks like the exact solution[21].

The model I intend to make will also consist of differential equations, and therefore I will be using Euler's method to help me solve it. I will be building and solving my model with the help of the program Spyder, which is perhaps more commonly known as Python. Spyder can quickly do all the calculations related to Euler's method. Solving them yourself would leave you an enormous amount of work when you choose a small Δt interval, as you would have to calculate a large amount of points on your graph.

The Great Barrier Reef

<u>Introduction</u>
The Great Barrier Reef is most commonly known for its coral. Tourism is especially attracted by this, and the industry around it generated 4.228 billion dollars just in 2003[10]. Because of the beauty of the coral the reef is a UNESCO World Heritage Site[23]. Nowadays they are still in a spotlight, but that is because the reef is not doing so well[7]. The size of the reef is about 344,440 square kilometres[23], and it has lost about half of it's coral cover in the last couple decades[7]. People suspect that this will continue if we do not do something to protect the reef. The image below illustrates what has happened in the last decades.

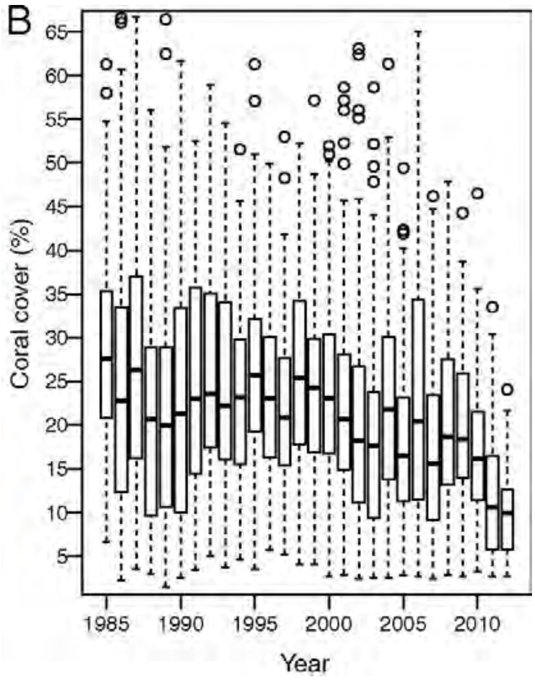

Figure 5: Box plot of the percentage of the reef covered by live coral in recent years[7].
The white boxes are the inner 50% (one box 25%) of measurements, the dotted lines are the outer measurements, the black line between the boxes is the average measurement.

<u>Coral and algae + symbiosis</u>
Corals are animals, despite their inability to perform locomotion (to move from one place to the other). They consist of coral polyps[5], which excrete a kind of calcium that forms the limestone skeleton on which they live. That skeleton together with the corals is considered the coral reef. Corals can be divided into 2 different kinds: hard and soft corals.

Hard corals are the corals that have a limestone "house". They form the basis of the reef and allow the other corals to grow on top of them. The soft corals do not have a hard limestone skeleton, and appear more like plants. The jelly-ish branches provide a safe haven for many other marine species. Yet, these branches also serve as a food source for them. The coral prevents itself from being eaten by making chemical substances that make it distasteful and even poisonous to some of those animals.

Corals are animals and need a way to acquire food[24]. They hunt for tiny fish or plankton or get their energy from the symbiosis they have with the algae Zooxanthellae. This algae houses itself in the corals tissues and performs photosynthesis, creating sugars in the process. The coral receives some of these sugars from the algae, and therefore the algae provide the coral with a food source.

Reproduction

Corals can reproduce themselves in two ways: asexual and sexual. The asexual variant means that the coral divides itself into two parts, forming a genetically identical new animal. Sexual reproduction, also known as coral spawning, often takes place around the full moon in November. The corals release eggs and sperm in the water over the course of a couple days. The eggs get fertilised, and can settle at the ocean beds to create new coral polyps. Not all of the eggs survive, as they are a food source for some other animals.

Bleaching

Zooxanthellae also gives coral their colour. When corals are under stress, which might be caused by rising sea temperatures (which in turn is caused by carbon pollution), they will eject the algae out of their tissues. This is known as coral bleaching, and is one of the many threats to the coral reefs around the world[4]. When the coral ejects the algae it also ejects one of it’s food sources, which might harm the animal in the future. Therefore bleaching should be prevented.

Yet, despite bleaching and other minor damage causes, the coral cover would increase by about 0.89% per year[7]. These other causes of damage are mainly storms. During a storm the skeleton of some corals can be damaged, which might lead to death. However, an increase in temperature will increase the amount of coral bleaching, and reduce the coral’s natural growth rate.

Predation by COTS

Another threat to coral reefs is a species that feeds on the corals - the Crown Of Thorns Starfish (abbreviated as COTS). One starfish can become up to 16 years old[1]. Large adults eat about 161 cm^2 of coral per day in winter and between 357-478 cm^2 per day in summer. Smaller starfish eat 66 cm^2 in winter and 155-234 cm^2 in summer per day[13]. A starfish on average eats about 6 m^2 per year[22].
During outbreaks they can quickly destroy reefs. Controlling their outbreaks might be the most important part of saving the reef.

The starfish eat corals by extruding their stomach over the coral polyps and releasing enzymes that digest them. the remains will be absorbed and used as food by the starfish[22].

When the temperature increases the larvae of the starfish have increased survival rate (1-2 degrees increases the rate by 240%)[2]. The rise in temperatures of the sea level is therefore also something that needs to be prevented to tackle the starfish outbreaks. Studies have shown that at least 10% of the reef needs to be covered with live coral for the starfish population to be able to grow[6], and that 99% of the newly born starfish die within 2 years[20].

The starfish can release up to 50 million eggs every year[2]. Not all of them will be fertilised every single spawning season, but they can be very rapid breeders nonetheless.

A mathematical model of the Great Barrier Reef

Because there is an infinite amount of numbers and variables, there will technically also be an infinite amount of possible models that can be made. These can be about specific things about the Great Barrier Reef, or might try to describe the reef as a whole. My model will be one of all the possibilities. Since the main concern is the coral cover of the reef my model will try to describe that.

I will be making this model in Python since I have more experience with this program, and with Python I can easily make graphs. I do have experience with other software like coach, but they do not provide me with the right tools for this job.

The main thing I want my model to be able to tell me is what will happen on the Great Barrier Reef. I don’t care about getting the numbers as close to reality as possible because that does not help in our understanding of the reef. If my model accurately describes reality, it’s always a plus, but I care more about the behaviour between the parameters than the precise values belonging to them.

Modeling cycle 1

Problem phase:

As I have mentioned earlier several times already, the Great Barrier Reef is not doing very well. The coral cover is declining, but as far as I am concerned we don't know how fast. This is what I consider the problem, and problems have to be solved.

According to the Australian Institute of Marine Science[7] the coral cover would have a growth rate of 0.89% annually in the absence of Crown-of-Thorns starfish. This rate already contains the effects bleaching and storms have on the coral.

Modeling phase:

To start with creating the model I will need something to represent the coral in the equation. In the model we will define C(t) as the size of the coral cover on the reef at any given time 't' in square kilometres. Since I plan to be looking at the coral cover over a few decades it would be best to measure the time in years. Hence 't' is in years.

The coral cover changes over the course of time. Imagine we looked at the coral cover a year ago. The new cover would be the same as the cover a year ago + the difference in coral cover. In maths that means the following:

$$C(t+\Delta t) = C(t) + \Delta C$$

In this story Δt is a small time difference, imagine it is about a minute. About ΔC I know that the coral cover increased by 0.89% annually. This means that coral cover difference is $0.0089C(t)$ over the same period Δt, the change over that certain time period is then described by $0.0089C(t)\Delta t$

If I replace ΔC with $0.0089C(t)\Delta t$ I get the following:

$$C(t+\Delta t) = C(t) + 0.0089C(t)\Delta t$$

Since I am accustomed to using differential equations in models I will be converting this model to a differential equation. Let's start by moving the C(t) to the left side by subtracting it from both sides of the equation. This results in the following:

$$C(t+\Delta t) - C(t) = 0.0089C(t)\Delta t$$

I do not like having 2 terms on the left side, so I will use the line below from before to get rid of those 2 terms.

$$C(t+\Delta t) = C(t) + \Delta C$$

If I subtract C(t) on both sides of this equation it results in the next equation:

$$C(t+\Delta t) - C(t) = \Delta C$$

If I translate the math back to English, it means nothing more than that the coral cover difference is the later measurement minus the earlier measurement, and that is logical and correct.

Take a look at what I know by now:

$$C(t + \Delta t) - C(t) = \Delta C$$

$$C(t + \Delta t) - C(t) = 0.0089C(t)\Delta t$$

The left side of both equations are the same. That in turn means the right sides must be the same as well. Combining both equations results in the following, single equation:

$$\Delta C = 0.0089C(t)\Delta t$$

A differential equation is used to describe a speed at which something grows or declines. The speed here is the same as the difference in coral cover divided by the time in which that difference was created. Because I want to get a differential equation I will divide both sides by Δt.

$$\frac{\Delta C}{\Delta t} = 0.0089C(t)$$

And I have finally gotten the differential equation that I was looking for. Δt In this case is infinitely small. Do note that the growth rate of the coral is constant, which means I have assumed climate change and rising sea temperatures do not play a role in the model. The impact bleaching has is thought of to be small[7], but not non-existent

With Euler's method you can calculate the next point on your graph if you have a starting point. However, I do not have a starting point yet. I still need to find C(0), the coral cover at time t = 0.
According to an article I found on the internet[7]. The mean coral cover in 1985 was 28.0% of the Great Barrier Reef. Do note that this is not the average, but the most measured number on all sites, making it close to the average I need.
Since the Great Barrier Reef is 344,440 square kilometres[23], the coral cover at time t = 0 should be 0.28 x 344,440 = 96,443.2 square kilometres.
I have decided to take t = 0 in 1985 since the article provided the coral cover from 1985 until 2012. This way I have a means to check if my model is accurate.

To bring this modeling phase to an end, I have gotten the following model:

$$\frac{\Delta C}{\Delta t} = 0.0089C(t)$$

$$C(0) = 96,443.2$$

With t =0 in the year 1985, C expressed in square kilometres and t in years.

Calculation phase
After the implementation in Python I ran the model, which resulted in the following graph.

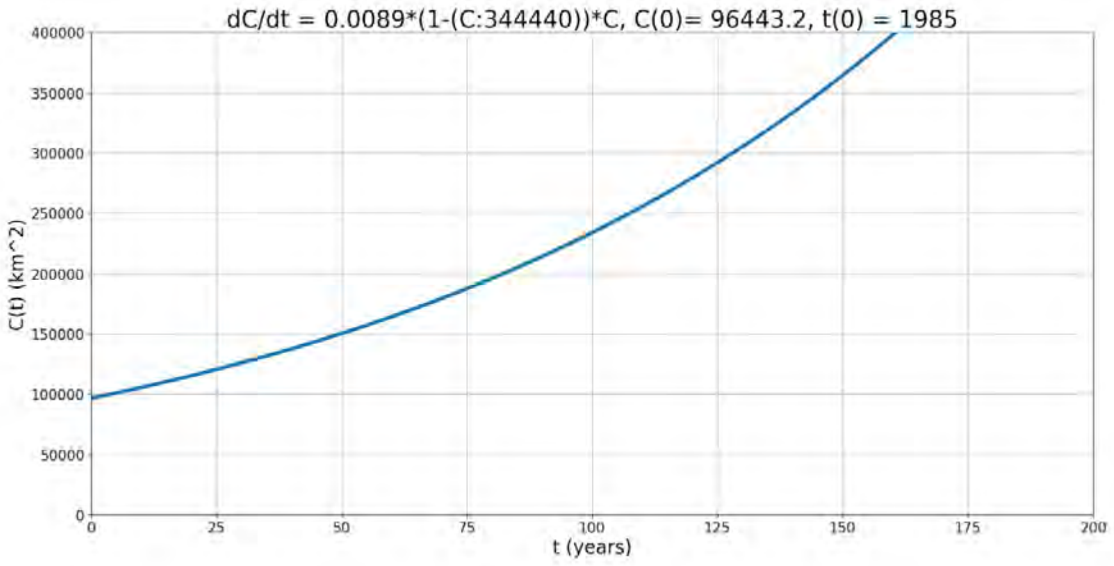

figure 6: predicted coral cover after cycle 1

There is not a whole lot of things to say about it, apart from that the coral cover would keep rising indefinitely at an ever increasing speed.

Verification phase
However, the Great Barrier Reef had a size of 344,440 kilometres squared[23]. This means that the maximum coral cover cannot be larger than the 344,440 kilometres squared which the reef covers. In the graph you can see that after roughly 145 years the coral cover does get larger than the reef itself. And as I have tried to make clear, that is impossible. The model therefore is not accurate enough (as models after 1 cycle usually are)

This means that the model lacks a factor that limits the growth of the reef, and that will have to be added in the next cycle.

Modeling cycle 2

Problem phase

As I mentioned in the previous cycle's verification phase, the coral cover cannot grow indefinitely. However, In the current model it can. This means that I will have to add a factor that limits the coral cover's growth and does not allow it to surpass the total size of the Great Barrier Reef, which is 344,440 kilometres squared[23].

So how am I going to limit the growth of the coral in the model?

Modeling phase

In order to limit the growth I will have to add an inhibiting factor to prevent the coral from being able to grow larger than the reef itself. The inhibiting factor in our model will be:

$$(1 - \frac{C_t}{344440})$$

This will have to be added in the same term as the growth rate in the model, because it influences the coral's growth. The new model with the inhibiting factor will be the following:

$$\frac{\Delta C}{\Delta t} = 0.0089(1 - \frac{C_t}{344,440})C_t$$

$$C(0) = 96,443.2$$

and t = 0 in the year 1985 with t expressed in years and C in square kilometres.

The way this inhibiting factor works is something I want to mention, because it causes the growth in this model to be logistical. This means it increases in the beginning, but as soon as it gets closer to a certain value the growth decreases.

In this model the maximum* value for C(t) is 344,440 kilometres squared. As soon as C(t) gets close to this value the inhibiting factor becomes close to 0, therefore decreasing the growth and eventually the growth will be so small that it can simply be ignored.

note: since I am using Euler's method it can be larger than this maximum value if* ***Δ**t isn't small enough!*

Calculations Phase

After Implementing the model in Python and running it, the following graph was created.

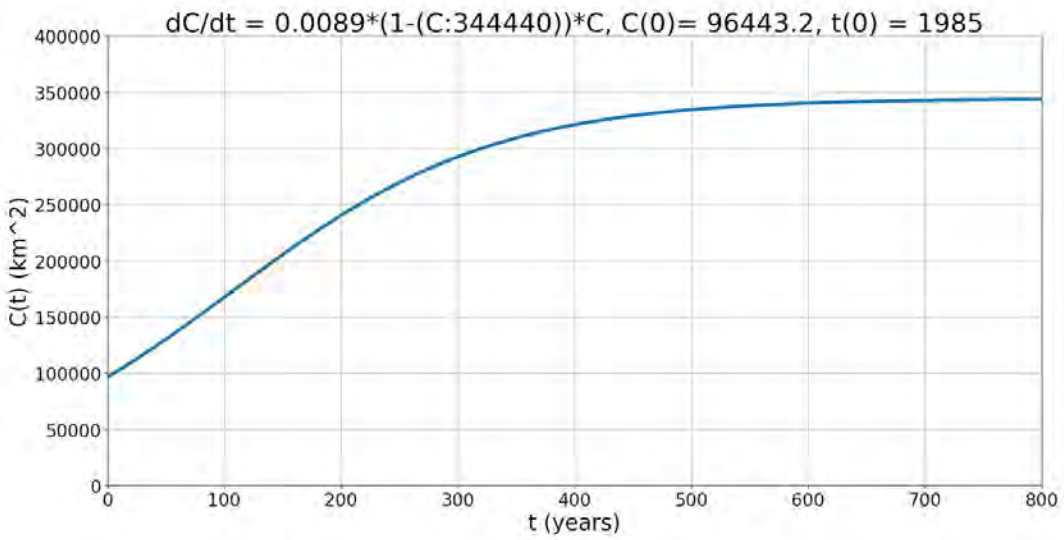

Figure 7: predicted coral cover after cycle 2

As I mentioned earlier in the modeling phase, the inhibiting factor will eventually be reduced to a number so small that the general growth does the same. This results in a stop in the increase of reef covered by coral - a point of equilibrium. The graph reaches this point at about t = 650 (mathematically, the solution will never reach this point).

The mathematical reason for this has been put below.
I start with the model I currently have.

$$\frac{\Delta C}{\Delta t} = 0.0089(1 - \frac{C_t}{344,440})C_t$$

In the point of equilibrium the growth of the reef is 0, so the right hand side of this equation should equal 0 as well.

$$0 = 0.0089(1 - \frac{C_t}{344,440})C_t$$

This equation is of the type: 0 = AB with A being 0.0089C and B being the inhibiting factor. This means either A or B = 0, so I get the following 2 equations.

1. $0 = 0.0089C_t$

or

2. $$0 = (1 - \frac{C_t}{344,440})$$

Solving the first equation gives the solution C = 0, this means that there also is a point of equilibrium when there is no living coral in the reef. If you think about it is is logical, when everything is dead the species can't reproduce and cannot grow in population.

Mathematically this is a very easy answer, as most models often have a point of equilibrium in a so called 0-vector. mathematicians often do not accept this as a point of equilibrium, while it technically is one.

The second equation can be solved with an extra step:

$$1 = \frac{C_t}{344,440},$$

yielding the solution C(t) = 344,440. The total size of the Great Barrier Reef. Which is also logical, because coral cover cannot exceed the area it can grow on. This solution also means that the inhibiting factor I implemented this cycle has been effective at limiting coral growth.

<u>Verification Phase</u>

The growth of the coral has successfully been limited to not surpass the size of the reef. However, coral is not the only species in the Great Barrier Reef. The Crown Of Thorns Starfish feeds on coral polyps. The model currently does not include the starfish, while they are a core part of the Great Barrier Reef and influence the live coral cover of it. The next cycle will have to add them.

Modeling cycle 3

<u>Problem Phase</u>

The problem I am currently facing with our model is that it does not yet include all species of the reef. I mentioned earlier that the starfish need a live coral cover of 10% of the total reef, which is equal to 34,444 kilometres squared covered in coral[6].

For this cycle I am going to be introducing the new variable of S (t), which is the number of starfish living in the reef at any given time t.

<u>Modeling Phase</u>

I have been trying to come across the amount of starfish living in the reef in 1985, but I couldn't find the data. However, I have found the density of the starfish in 1986. I will have to change the starting condition of the coral species to match this year.
C(0) will therefore become 0.225 (see the graph below) * 344,440 = 77499 kilometres squared[7].

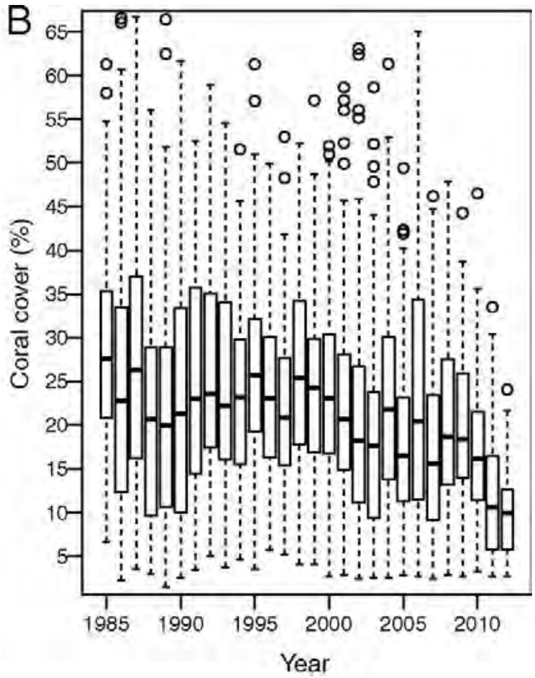

Figure 5: Box plot of the percentage of the reef covered by live coral in recent years[7].
The white boxes are the inner 50% (one box 25%) of measurements, the dotted lines are the outer measurements, the black line between the boxes is the average value.

According to the AIMS the density of the starfish population in 1986 was 0.25 per 2-minute Manta Tow[3]. A Manta Tow is a method they use to gather data in the Great Barrier Reef. A diver travels with a boat along the edges of a reef whilst attached to a manta board. The boat travels at 4 km/h and the diver is supposed to observe a width of 10m during the 2 minutes[14].

This means that the boat travels 4000 metres per hour, which is equivalent to 133.33… metres per 2 minutes, or 0.13333... kilometres per 2 minutes. The diver observes a width of 10 metres, or 0.01 km. This results in an observed area of 0.0013333… kilometres squared.

This means that the starfish density is 0.25 per 0.0013333… squared kilometres. that is 187.5 starfish every square kilometre.
Because the Great Barrier Reef has a surface of 344,440 square kilometres[23], there were a total of 344,440 * 187.5 = about 64582502 starfish present.
S(0) is therefore 64582502.

Because the number of starfish is so much larger than the coral cover, it is hard to make a good graph. I will change the unit in which we express C(t) to square metres. This also means that I will change some numbers in certain factors, for example the inhibiting factor from cycle 2, as they are also based on square kilometres.

The only thing that I need is a equation for the growth of the starfish population. For this function I will make use of the fact that juvenile starfish need 10% live coral cover[6] (34444 square kilometres of coral) to grow. This sounds like the opposite of an inhibiting factor, as it should increase after a certain threshold. The differential equation below will only be positive if C(t) is bigger than 34444 km squared, and growth will only exist when the equation is positive.

$$\frac{\Delta S}{\Delta t} = (\frac{C_t}{34,444,000,000} - 1)S_t$$

We now have a differential equation for both important species, and the entire model will be put below.

$$\frac{\Delta C}{\Delta t} = 0.0089(1 - \frac{C_t}{344,440,000,000})C_t$$ With

$$C_0 = 77499000000$$

$$\frac{\Delta S}{\Delta t} = (\frac{C_t}{34,444,000,000} - 1)S_t$$

With $S_0 = 64582502$

And t = 0 in the year 1986, C expressed in squared metres, S expressed in animals (since animals is not an official unit it is mathematically seen as dimensionless) and t expressed in years.

Calculations Phase:
Running this model with the help of Python in my own computer, I got the following results:

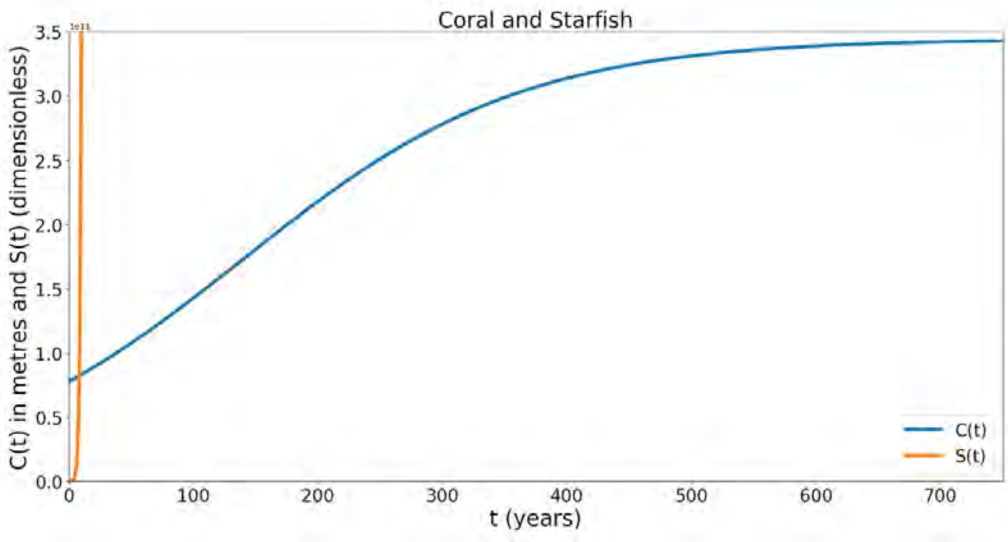

Figure 8: predicted coral and starfish cover after cycle 3

The predicted coral population should not have changed since the last cycle. Because the coral cover is greater than 10% of the reef at the start the population of starfish will grow indefinitely.

Verification phase
After several cycles I like to go a bit more in-depth in determining the model's characteristics. However, I have decided not to do that in this cycle. As you can see, the population of starfish grows extremely quickly. My data suggests that they can be very rapid breeders, but this is too rapid. I have not yet added to my model other than that the starfish need 10% of live coral cover to grow[6]. My model doesn't describe their reproduction. The next cycle will have to add them.

Furthermore, my model currently doesn't include feeding behaviours of the starfish. This also has a severe impact on both species' growth, as the coral population should get eaten by the starfish. When that happens, there will be less coral to feed on and the starfish will die since they need 10% live coral cover[6].

Modeling cycle 4

<u>problem phase</u>
As I mentioned in last cycle's verification phase, there is no growth rate for the starfish as of yet. This cycle I will be adding the growth rate based on the reproduction of the starfish to the differential equation related to the animals.

<u>Modeling phase</u>
On the internet I have found several articles with information about the growth rate of the starfish. It turns out that 99% of the starfish that are recruited, a fancy word for born, died within 2 years[20]. This means that 10% of the starfish survive the year (0.1 * 0.1 = 0.01). Do note that this was measured on a different reef, but since I am talking about the same animal the percentage could be used for any other coral reef.

Another article[15] stated that the fertilisation rate of the eggs released during the spawning season appears to decrease by 50% every 30 metres. Based on this and the amount of eggs a female starfish produces (which is up to 50 million[2]) I can calculate how many new starfish will be born in a certain amount of time. These newly recruited starfish will be the amount of eggs that have been fertilised during the spawning season. In order for us to calculate this I am going to divide the reef into square blocks, populated each by one starfish as shown below. I will assume that half of the starfish population is female and half is male

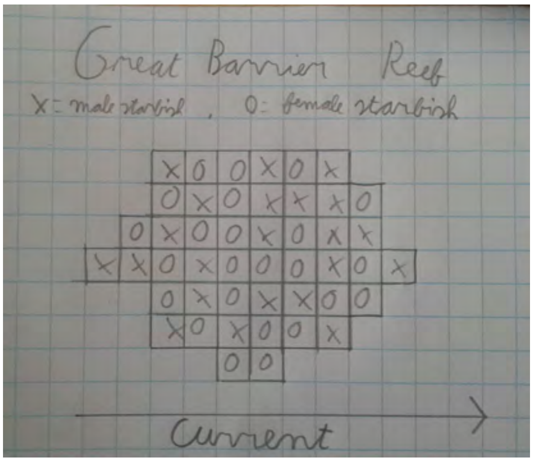

Figure 9: visualisation of the division of the reef

The starfish reproduce by releasing their eggs and sperm in the water. When an egg cell meets a sperm cell the egg cell is fertilised and will result in the creation of a new starfish. Because the eggs and sperm are released in the water, the water current is what determines where the eggs will go. This current is represented by an arrow. To keep things easy, I am going to assume that the current is always parallel to a side of a square. If for some reason the current would change, I would rearrange the squares with the starfish in such a way that the current would be parallel again.

To simplify even more I will assume that when a female starfish releases her eggs only the first two starfish in proximity in the direction of the ocean current will be able to fertilise the eggs. This situation with it's possible combinations has been put below. Do note that more starfish might be able to fertilise in reality.

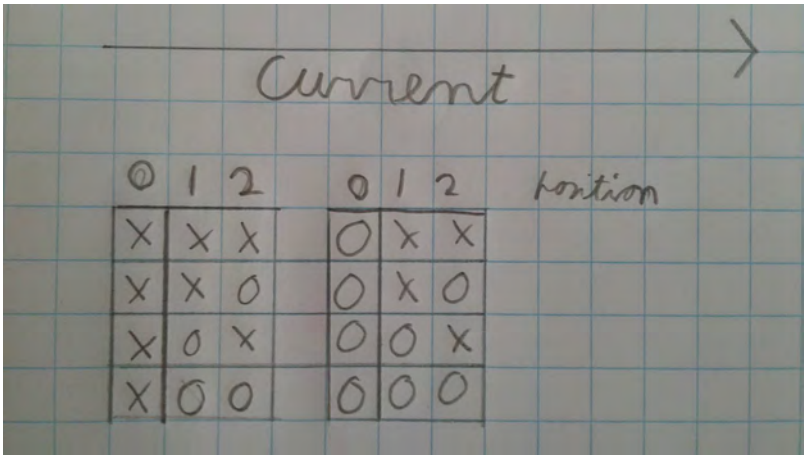

Figure 10: possible combinations with up to 2 starfish behind the first one

Position 0 is the starfish that will release the eggs in the water, position 1 is the first to have a chance to fertilise them and position 2 is the second to have a chance to fertilise them.

As you can see, only half of the starfish on position 0 is able to release eggs in the water. Because half of the starfish population is male. Only half of the egg loads (I am not talking about the quantity of eggs) will undergo fertilisation on both positions 1 and 2.

The data I found on the internet suggested that every 30 metres the fertilization rate is halved. This "fertilization factor" for position 1 can be translated to:

$$2^{-\frac{d}{30}}$$

with d being the distance between the 2 starfish. This distance is equal to the square root of the square area 1 starfish occupies. Because 50% of the starfish population is male there is 50% chance that the eggs will be fertilised on position 1. All of this translates the fertilisation factor to the following:

$$\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{St}}}{30}}$$

Because the distance between the starfish on positions 0 and 2 is twice as far as the distance between the starfish on positions 0 and 1. The fertilisation factor for the starfish on position 2 becomes

$$\frac{1}{2} * 2^{-\frac{2\sqrt{\frac{344440000000}{St}}}{30}}$$

The total fertilisation rate will be equal to the sum of the eggs fertilised on position 1 and on position 2. The total fertilisation factor therefore is:

$$(\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S_t}}}{30}} + \frac{1}{2} * 2^{-\frac{2\sqrt{\frac{344440000000}{S_t}}}{30}})$$

However, there is a problem with this fertilization factor. When the egg loads go from position 1 to position 2, some of them are fertilised. At position 2 they can be fertilised again. Biologically this second fertilisation would be impossible and would only result in 1 starfish. But Mathematically speaking it is not impossible and will result in 2 new starfish from 1 single fertilised egg. There is a way to circumvent this, which is by saying fertilised eggs cannot be fertilised again. But this would give the sperm cells of the starfish highly sensitive cognitive functions because they would be able to determine which egg cell is fertilised and which one is not. That is biologically incorrect as well.

WIth this fertilisation factor and that a female starfish produces up to 50 million eggs. I can finally finish the equation for the starfish. Half of the population of starfish is female, which means half of the starfish produce 50 million eggs. The total equation would therefore be:

$$\frac{\Delta S}{\Delta t} = (\frac{C_t}{34444000000} - 1) * 50000000 * (\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S_t}}}{30}} + \frac{1}{2} * 2^{-\frac{2\sqrt{\frac{344440000000}{S_t}}}{30}}) * \frac{1}{2}S_t$$

(The 50 million and the 0.5 can be simplified to 25 million:)

$$\frac{\Delta S}{\Delta t} = (\frac{C_t}{34444000000} - 1) * 25000000 * (\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S_t}}}{30}} + \frac{1}{2} * 2^{-\frac{2\sqrt{\frac{344440000000}{S_t}}}{30}}) * S_t$$

All that is currently missing in this model is the fact that 10% of the starfish survive the year. I will need to add a factor of 0.1 to the fertilisation factor, which means that only 2.5 million unfertilised eggs will survive the year. The result is the following:

$$\frac{\Delta S}{\Delta t} = (\frac{C_t}{34444000000} - 1) * 2500000 * (\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S_t}}}{30}} + \frac{1}{2} * 2^{-\frac{2\sqrt{\frac{344440000000}{S_t}}}{30}}) * S_t$$

Do note that the starfish produce up to 50 million eggs, and I have stated they always produce 50 million. I have left an 0.5 inside the brackets within the fertilization factor because this is useful for showing only up to 100% of eggs can be fertilised.

The equation for the coral cover has remained unchanged. This means the model for cycle 4 will be as follows:

$$\frac{\Delta C}{\Delta t} = 0.0089(1 - \frac{C_t}{344,440,000,000})C_t$$

$$C_0 = 77499000000$$

$$\frac{\Delta S}{\Delta t} = (\frac{C_t}{34444000000} - 1) * 2500000 * (\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S_t}}}{30}} + \frac{1}{2} * 2^{-\frac{{}^{2}\sqrt{\frac{344440000000}{S_t}}}{30}}) * S_t$$

$$S_0 = 64582502$$

With t = 0 in the year 1986, C expressed in squared metres, S dimensionless (for clarity, I consider it to be expressed in animals) and t expressed in years.

<u>Calculations phase</u>

Running the model in Python has resulted in the following graphs.

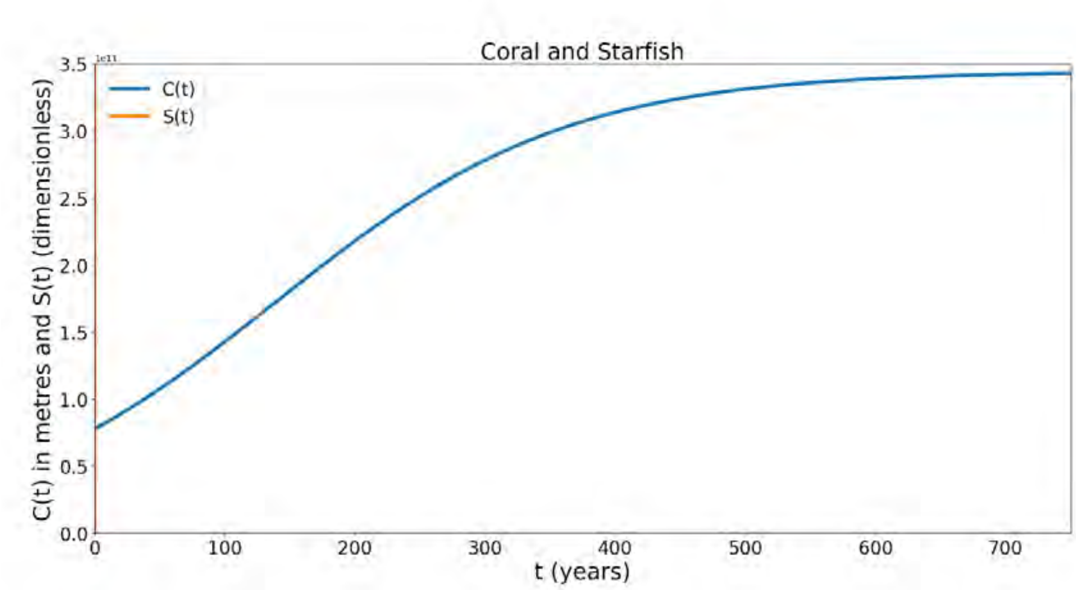

Figure 11.1: predicted coral and starfish populations after cycle 4. The starfish population grows so fast that it is almost the same line as the vertical axis.

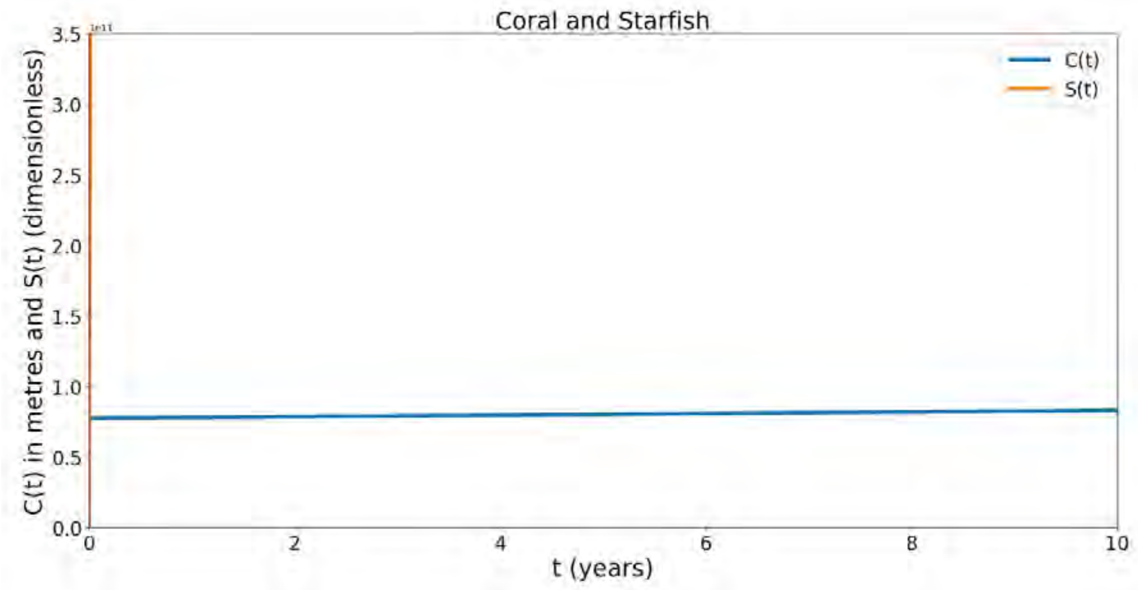

Figure 11.2: predicted coral and starfish cover after cycle 4

<u>Verification phase</u>

The only thing that has really changed is the speed at which the starfish population grows. But the reasons I mentioned in cycle 3 for the model not working are present here as well. Mainly because the starfish still don't eat the coral. I will be adding this in the next cycle.

Modeling cycle 5

<u>Problem phase</u>
As I mentioned in the last verification phase, I have 1 important piece of missing information: the fact that starfish eat coral. Based on the articles I found on the internet I will include the feeding behaviour of the starfish into the model. Large adults eat about 161 cm² of coral per day in winter and between 357-478 cm² per day in summer[13]. This is 0.0161 m² in winter and 0.0357-0.0478 m² in summer. The average of the feeding in summer is 0.04175 m².

It will be important in the modeling phase that when we celebrate new years eve, it is winter in the northern hemisphere of the earth, but summer in the southern hemisphere. It is therefore summer at the start of the new year in Australia.

<u>Modeling phase</u>
Because I want to include the fact that the starfish eat more in the summer than in the winter I will be using either a cosinus or a sinus. This is to put the harmonical change of the seasons in the model, and that change influences the feeding behaviour.

The difference in the amount of coral the starfish eat in summer and winter is 0.04175 - 0.0161 = 0.02565 m². This will be the factor in front of the cosinus, as this will have to be added or not added.

SInce the year begins with summer in australia, and the cosine of 0 = 1, I will have to use a cosinus in our feeding factor. Combining this with all of our other information I can get to the following feeding factor factor:

$$-(0.0161 + |0.02565 * \cos(\Pi * t)|) * S_t$$

The minus in front represents the fact that the coral gets eaten. Because the cosine of pi = -1, and I don't want to have the starfish spit out the coral they ate, I have put the cosinus in absolute values. I want the period for this cosinus to be 1 year. To do this I should have added 2 * pi before the t in the cosinus, but because of the absolute value sign I am only talking about half of the exact-value-circle, which I move on every year. Hence I only used half of the 2 * pi. The S(t) behind the feeding factor represents that every starfish feeds on the coral.

Adding this to the total equation of the coral cover, I get the following result:

$$\frac{\Delta C}{\Delta t} = 0.0089(1 - \frac{C_t}{344440000000}) * C_t - (0.0161 + |0.02565 * \cos(\Pi * t)|) * S_t$$

After corresponding with one of my tutors from the Junior TU Delft course, Dennis den Ouden, I received a few remarks about the model. These changes mainly included changing some symbols to the correct version. In specific the following changes were suggested: change the Delta (triangle) to d, change the capital Pi into a lower case Pi. It would also be possible to remove the multiplication marks (*) between the terms if it is clear one should be there. The subscript t could also be removed.

The equations from my model will then look as follows:

$$\frac{dS}{dt} = (\frac{C}{34444000000} - 1)2500000(\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} + \frac{1}{2} * 2^{-\frac{\sqrt[2]{\frac{344440000000}{S}}}{30}})S$$

$$\frac{dC}{dt} = 0.0089(1 - \frac{C}{344440000000})C - (0.0161 + |0.02565\cos(\pi t)|)S$$

Another more important thing I got notified of is that the starfish currently do not die of age, only of food shortage. This means I will be adding a natural cause of death to the model. After doing some research on the internet I found out that the starfish's life expectancy is about 16 years[1]. Assuming that the age distribution in the starfish population is even, meaning there are equal amounts of starfish that are 2 year old compared to 10 year old starfish, I should be able to say that 1/16 of the starfish population dies every year. Adding this to the model I got the following:

$$\frac{dS}{dt} = (\frac{C}{34444000000} - 1)2500000(\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} + \frac{1}{2} * 2^{-\frac{\sqrt[2]{\frac{344440000000}{S}}}{30}})S - \frac{1}{16}S$$

$$\frac{dC}{dt} = 0.0089(1 - \frac{C}{344440000000})C - (0.0161 + |0.02565\cos(\pi t)|)S$$

with the initial values of $S_0 = 64582502$ and

$C_0 = 77499000000$

with t = 0 in the year 1986, C expressed in squared metres, S expressed in animals (although technically dimensionless) and t expressed in years.

<u>Calculations phase.</u>

Running the model in Python gives me the following graph. I have added the Python code in the appendix in case you want to see what it looks like.

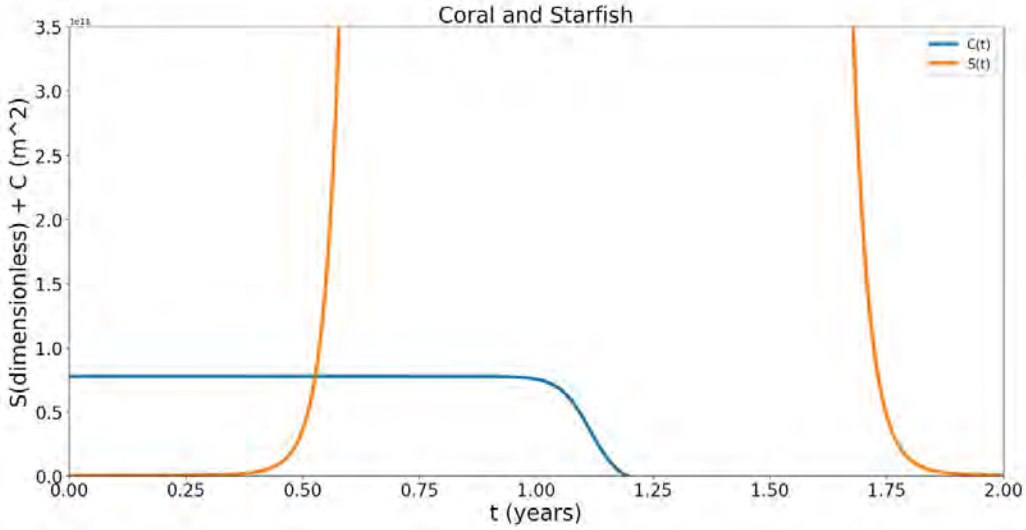

Figure 12: predicted coral and starfish populations after cycle 5. Do note that the speed of the differential equations in this graph has been reduced by 100000 times in order to properly get an image I can talk about.

The relation between predator and prey does seem to be working. Because the amount of coral is high in the beginning, the starfish population will receive the ability to grow. And because the starfish population increases, the coral population will start to decrease. Because the growth of the starfish is so quick it will wipe out all of the coral on the Great Barrier Reef.

Because I am in my final modeling cycle I will start to calculate the points of equilibrium and the corresponding eigenvalues. This should give me insight into the future condition of the reef.

In the points of equilibrium the values of S and C do not change when I take another timestep Delta t. This means the differential equations should equal zero. The respective equations are:

$$0 = \left(\frac{C}{34444000000} - 1\right)2500000\left(\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} + \frac{1}{2} * 2^{-\frac{\sqrt[2]{\frac{344440000000}{S}}}{30}}\right)S - \frac{1}{16}S$$

and

$$0 = 0.0089\left(1 - \frac{C}{344440000000}\right)C - (0.0161 + |0.02565\cos(\pi t)|)S$$

To simplify them I will replace the fertilisation factor

$$\left(\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S_t}}}{30}} + \frac{1}{2} * 2^{-\frac{\sqrt[2]{\frac{344440000000}{S_t}}}{30}}\right)$$

with the letter F. I will also bring the S outside of the brackets. This results in the following equation.

$$0 = ((\frac{C}{34444000000} - 1)2500000F - \frac{1}{16})S$$

From here you can find 2 solutions: S = 0 or

$$0 = (\frac{C}{34444000000} - 1)2500000F - \frac{1}{16}$$

Bringing the -1/16 to the left side, I get the following:

$$\frac{1}{16} = (\frac{C}{34444000000} - 1)2500000F$$

Then I will divide both sides by 2500000F,

$$\frac{1}{40000000F} = \frac{C}{34444000000} - 1$$

Next I will bring the 1 to the right

$$\frac{1}{40000000F} + 1 = \frac{C}{34444000000}$$

To finally find the answer for the equilibrium value of C, I will multiply both sides by 34444000000

$$C = \frac{861.1}{F} + 3444000000$$

We do not yet know the precise value of C, because it is expressed in F. This is an issue that I will solve later.

<u>The differential equation for the starfish have resulted in the following 2 solutions:</u>

$$C = \frac{861.1}{F} + 3444000000$$

$$S = 0$$

Using S = 0 in

$$0 = 0.0089(1 - \frac{C}{344440000000})C - (0.0161 + |0.02565\cos(\pi t)|)S$$

gives the following equation:

$$0 = 0.0089(1 - \frac{C}{344440000000})C$$

I will divide both sides by 0.0089

$$0 = (1 - \frac{C}{344440000000})C$$

From this you can get the following 2 solutions: C = 0, or

$$0 = 1 - \frac{C}{344440000000}$$

Then I bring the C:344440000000 to the left

$$1 = \frac{C}{344440000000}$$

Multiplying both sides with 344440000000 gives

$$C = 344440000000$$

Therefore, filling in S = 0 results in the following 2 points of equilibrium:

$$C = 0, S = 0$$

$$C = 344440000000, S = 0$$

Using $C = \frac{861.1}{F} + 34444000000$ in

$$0 = 0.0089(1 - \frac{C}{344440000000})C - (0.0161 + |0.02565\cos(\pi t)|)S$$

gives the following equation:

$$0 = 0.0089(1 - \frac{\frac{861.1}{F} + 34444000000}{344440000000})(\frac{861.1}{F} + 34444000000) - (0.0161 + |0.02565\cos(\pi t)|)S$$

I'll start with simplifying the first term in between brackets.

$$0 = 0.0089(1 - \frac{861.1}{34444000000F} - 0.1)(\frac{861.1}{F} + 34444000000) - (0.0161 + |0.02565\cos(\pi t)|)S$$

removing the 0.1 from the 1 gives:

$$0 = 0.0089(0.9 - \frac{861.1}{34444000000F})(\frac{861.1}{F} + 34444000000) - (0.0161 + |0.02565\cos(\pi t)|)S$$

finishing the multiplication from the 0.0089 with the first brackets gives

$$0 = (0.00801 - \frac{7.66379}{34444000000F})(\frac{861.1}{F} + 34444000000) - (0.0161 + |0.02565\cos(\pi t)|)S$$

Multiplying all the brackets with each other and simplifying them gives the following:
Do note that I have rounded off until 3 decimals.

$$0 = \frac{9.503 * 10^{19}F^2 + 2.112 * 10^{12}F - 6599.290}{344440000000F^2} - (0.0161 + |0.02565\cos(\pi t)|)S$$

Bringing the term with S to the left sides gives:

$$(0.0161 + |0.02565\cos(\pi t)|)S = \frac{9.503 * 10^{19}F^2 + 2.112 * 10^{12}F - 6599.290}{344440000000F^2}$$

Dividing by the factor in front of S gives:

$$S = \frac{9.503 * 10^{19}F^2 + 2.112 * 10^{12}F - 6599.290}{344440000000F^2(0.0161 + |0.02565\cos(\pi t)|)}$$

The “final” point/function of equilibrium is

$$C = \frac{861.1}{F} + 34444000000S, S = \frac{9.503 * 10^{19}F^2 + 2.112 * 10^{12}F - 6599.290}{344440000000F^2(0.0161 + |0.02565\cos(\pi t)|)}$$

This point of equilibrium is dependent of the time, since the cosinus contains a ‘t’. Because the starfish eat more in the summer, the equilibrium value of the starfish will decrease in the summer. It will increase in the winter, because they eat less and we need more starfish to keep the coral’s growth equal to 0. This means that we will have a point of equilibrium that moves along a trajectory.

There is one problem left: the point of equilibrium has been expressed in F, which also contains the variable S in its power. This technically means I do not yet know the point of equilibrium. I will have to find a way to get rid of the F in the equation.

I will start by finishing the division of the point of equilibrium of S:

$$S = \frac{9.503 * 10^{19}F^2 + 2.112 * 10^{12}F - 6599.290}{344440000000F^2(0.0161 + |0.02565\cos(\pi t)|)}$$

will be the same as

$$S = \frac{275897108}{0.0161 + |0.02565\cos(\pi t)|} + \frac{2.112 * 10^{12}F}{344440000000F^2(0.0161 + |0.02565\cos(\pi t)|)} + \frac{-6599.290}{344440000000F^2(0.0161 + |0.02565\cos(\pi t)|)}$$

Which can be simplified to

$$S = \frac{275897108}{0.0161 + |0.02565\cos(\pi t)|} + \frac{6.132}{F(0.0161 + |0.02565\cos(\pi t)|)} + \frac{-6599.290}{344440000000F^2(0.0161 + |0.02565\cos(\pi t)|)}$$

In the winter (t = ½ + k) the first term will be 439550362. Filling this in in F gives the value 0.399, meaning roughly 40% of the eggs will be fertilised. The second term of this fraction is still dependent on F. But F must have a minimum value of 0.399 .

Assuming I am dealing with a natural amount of starfish (whole from 0 till the positive infinity), the fertilisation factor will be between 0 (the limit of S goes towards 0 gives F = 0) and 1 (the limit of F goes towards infinity gives F = 1).

Because I know 0 < F < 1 and F > 0.399, I can say that 0.399 < F < 1.
This means the second term of the above equation will be up to 2.5x as large. The third term will be up to 6.25x as large. The difference in powers between the first term and the second and third might only be reduced by 1, which means the second and third term do not add much value and can be ignored.
This results in the point of equilibrium for S being approximately described by

$$S = \frac{275897108}{0.0161 + |0.02565\cos(\pi t)|}$$

And

$$C = \frac{861.1}{F} + 3444000000$$

In summer, the point of equilibrium is approximately:
S = 6608313964 and C = 34444001102

And in winter it is approximately:
S = 17136466340 and C = 34444001005

The Eigenvalues
Now that I know the points of equilibrium, I can determine how a solutions will behave when near those points. It is easy to determine this if I can write my model in a matrix-vector multiplication. But because I have a 2 with an S in its power this is impossible, and the matrix in the matrix-vector multiplication can be written as the jacobian matrix.

$$A = \begin{bmatrix} \left.\frac{\partial f_1}{\partial P}\right|_{(P_{\text{eq}},G_{\text{eq}})} & \left.\frac{\partial f_1}{\partial G}\right|_{(P_{\text{eq}},G_{\text{eq}})} \\ \left.\frac{\partial f_2}{\partial P}\right|_{(P_{\text{eq}},G_{\text{eq}})} & \left.\frac{\partial f_2}{\partial G}\right|_{(P_{\text{eq}},G_{\text{eq}})} \end{bmatrix}$$

Figure 13: the 2x2 jacobian matrix.

To find this matrix, I will need to determine the partial differentiated functions of both S and C differentiated towards both variables. With this and the points of equilibrium I can calculate what the jacobian matrix of our model would look like.

We will start with the easier function of the coral cover, which I will name f_1

$$f_1 = 0.0089(1 - \frac{C_t}{344440000000})C_t - (0.0161 + |0.02565\cos(\pi t)|)S_t$$

Differentiating this towards S yields the following result

$$\frac{\partial f_1}{\partial S} = -(0.0161 + |0.02565\cos(\pi t)|)$$

which in turn can be simplified to

$$\frac{\partial f_1}{\partial S} = -0.0161 - |0.02565\cos(\pi t)|$$

Differentiating f_1 towards C gives:

$$\frac{\partial f_1}{\partial C} = 0.0089(1 - \frac{C}{344440000000}) + 0.0089C(\frac{-1}{344440000000})$$

which we can simplify into

$$\frac{\partial f_1}{\partial C} = 0.0089 - \frac{0.0178C}{344440000000}$$

We will now take the function for the starfish population and name it f_2

$$f_2 = (\frac{C}{34444000000} - 1)2500000(\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} + \frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}})S - \frac{1}{16}S$$

Differentiating this towards C results in

$$\frac{\partial f_2}{\partial C} = (\frac{1}{34444000000})2500000(\frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} + \frac{1}{2} * 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}})S$$

which I can simplify until it becomes

$$\frac{\partial f_2}{\partial C} = (\frac{5}{137776})(2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} + 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}})S$$

Differentiating f_2 towards S gives a much larger equation than I would have anticipated.

$$\frac{\partial f_2}{\partial S} = (\frac{C}{34444000000} - 1)2500000(0.5 * 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} + 0.5 * 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}})$$

$$+(\frac{C}{34444000000} - 1)2500000 * 0.5 * S * (2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} * \frac{\ln 2}{2 * -30 * \sqrt{\frac{344440000000}{S}}} * \frac{-344440000000}{S^2}$$

$$+2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}} * \frac{\ln 2}{2 * -15 * \sqrt{\frac{344440000000}{S}}} * \frac{-344440000000}{S^2}) - \frac{1}{16}$$

Simplifying (by bringing the term with the C outside of brackets) gives:

$$\frac{\partial f_2}{\partial S} = ((\frac{C}{34444000000} - 1)1250000)((2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} + 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}})+$$

$$S * (2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} * \frac{\ln 2}{-60 * \sqrt{\frac{344440000000}{S}}} * \frac{-344440000000}{S^2} + 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}} * \frac{\ln 2}{-30 * \sqrt{\frac{344440000000}{S}}} * \frac{-344440000000}{S^2})) - \frac{1}{16}$$

Which can be simplified further into:

$$\frac{\partial f_2}{\partial S} = (\frac{5C}{137776} - 1250000)((2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} + 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}}) + S * (2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} * \frac{5740666667 \ln(2)}{S^2\sqrt{\frac{344440000000}{S}}} + 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}} * \frac{34444000000 \ln(2)}{3S^2 * \sqrt{\frac{344440000000}{S}}})) - \frac{1}{16}$$

Bringing the S in between the brackets gives:

$$\frac{\partial f_2}{\partial S} = (\frac{5C}{137776} - 1250000)((2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} + 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}}) + (2^{-\frac{\sqrt{\frac{344440000000}{S}}}{30}} * \frac{5740666667 \ln(2)}{S\sqrt{\frac{344440000000}{S}}} + 2^{-\frac{\sqrt{\frac{344440000000}{S}}}{15}} * \frac{34444000000 \ln(2)}{3S\sqrt{\frac{344440000000}{S}}})) - \frac{1}{16}$$

Now that I know the partially differentiated functions of both C and S both differentiated towards C and S, I can create our jacobian matrix. Using the point of equilibrium for summer gives:

$$\begin{bmatrix} 0.00712 & -0.04175 \\ 374766.25 & 0.00760 \end{bmatrix}$$

Figure 14: the jacobian matrix in summer

This means the following equation has to be solved in order to find the eigenvalues: (do note that in the calculations I have decided to round off some numbers)

$$(0.00712 - \lambda)(0.00760 - \lambda) - 374766.25 * -0.04175 = 0$$

We first get rid of the brackets.

$$0.000054112 - 0.01472\lambda + \lambda^2 + 15646.49 = 0$$

Then I add the terms not dependent on lambda and bring them to the right side.

$$\lambda^2 - 0.01472\lambda = -15646.49$$

We then build new brackets as follows.

$$(\lambda - 0.00736)^2 - 0.0000542 = -15646.49$$

We bring the -0.00000542 to the right (not that it will do much but I still have to).

$$(\lambda - 0.00736)^2 = -15646.49$$

This equation might seem unsolvable, but I can solve it with the help of the imaginary number.

$$(\lambda - 0.00736)^2 = 15646.49i^2$$

I then take the square root,

$$\lambda - 0.00736 = \pm 125.09i$$

and bring the -0.00736 to the right side

$$\lambda = 0.00736 \pm 125.09i$$

The only number I am currently interested in is the non-imaginary part of this number. The 0.00736. The fact that this is bigger than 0 means that the point of equilibrium is unstable, and that the solution will divert away from the point of equilibrium.

Calculating the jacobian matrix using the point of equilibrium for winter gives us the following matrix. (I have rounded off some numbers)

$$\begin{bmatrix} 0.00712 & -0.0161 \\ 1066227.3 & 0.00481 \end{bmatrix}$$

Figure 15: the jacobian matrix in winter

To find the eigenvalues, the following equation has to be solved

$$(0.00712 - \lambda)(0.00481 - \lambda) - 1066227.3 * -0.0161 = 0$$

Which can be written as

$$0.000034442 - 0.01193\lambda + \lambda^2 + 17166.26 = 0$$

Simplifying gives

$$\lambda^2 - 0.01193\lambda + 17166.26 = 0$$

Bringing the 17166.26 to the right and writing the left term as a multiplication of brackets gives

$$(\lambda - 0.00597)^2 - 0.0000356 = -17166.26$$

Simplifying and writing the right hand side with the imaginary number results in

$$(\lambda - 0.00597)^2 = 17166.26i^2$$

Then I will take the square root

$$\lambda - 0.00597 = \pm 131.02i$$

Bringing the -0.00597 to the right gives

$$\lambda = 0.00597 \pm 131.02i$$

Like summer, the non-imaginary part of this number is positive, which means the point of equilibrium is unstable and the solution of the model will move away from this point of equilibrium.

Because the point of equilibrium in both summer and winter is unstable, I am going to assume it is unstable all the time.

Plotting the results of the model in a phase diagram with C on the horizontal and S on the vertical axis, I got the following graph.

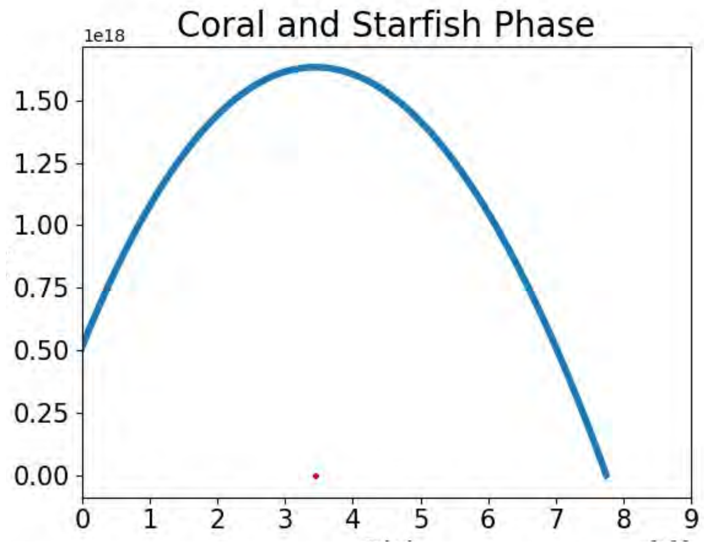

Figure 16: phase-diagram of the populations. The red dot is the point of equilibrium. The horizontal axis is C(t) expressed in squared metres. The vertical axis is S(t), which is dimensionless.

Verification phase

The predator-prey relationship now seems to be working. The only thing that still doesn't seem to be correct is the speed with which everything happens. The graph I gave in the calculations phase had the speed of both coral and starfish reduced by 100000 times, not reducing this speed would mean that everything on the reef would be dead a ridiculously short time. I didn't determine how short exactly, but imagine it would be in about a day. The cause for this is that I have assumed that the starfish always lay their maximum amount of eggs, which means their reproduction is slower than I assumed.

The unbelievable speed just means that in the conclusion I cannot say things about the time and quantity, but only about what happens. In other words: the only thing that is certain is the shape of the graph.

Out of simple curiosity I have run the model when the starfish only laid 10 eggs. The results, as shown in the graph below, indeed reduce the speed at which the starfish population grows to a more acceptable level. It however does not show any major changes in the shape of the graph. This means that my conclusion of only being able to say something about the shape of the graph is correct.

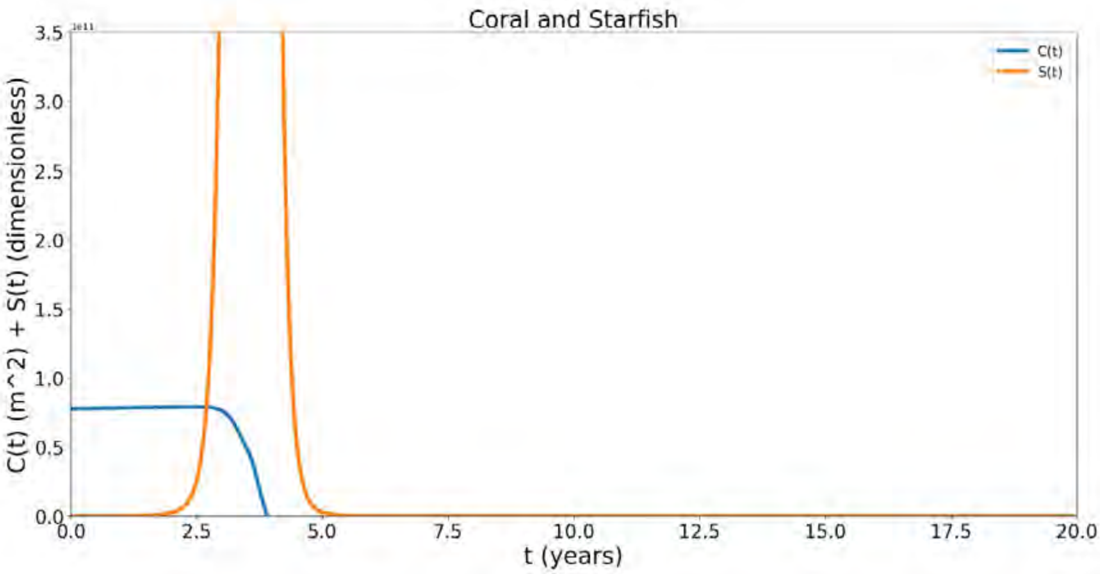

Figure 17: Result of model with a drastically reduced birth rate of the starfish population, not slowed 10000x.

Do note that during the creation of our model, I have made the following assumptions:
List of assumptions:

- The coral and starfish populations are spread out evenly across the reef
- The starfish always lay their maximum amount of eggs.
- Half of the starfish population is male, half is female.
- The age distribution in the starfish population is evenly spread.
- Only up to two starfish can fertilize the eggs released by a female specimen
- Only 2 animals exist on the reef. The Crown-of-Thorns Starfish is not at the top of the food chain, and individuals, in particular their eggs, are hunted by other predators, and in the model they are not hunted. This increases their population growth.
- Climate change is not present, as the coral's natural growth rate is constant. Sources suggest it is dependent on the temperature, but this is not the case in the model.

There are more minor assumptions, but I have not added these since they will hardly have any effect.

Conclusion

For this conclusion I am going to assume that the model right now is a perfect description of the Great Barrier Reef. This however is not the case, as new information and insights might become available the next decade, which I could not have used for my model. Furthermore, my own computer has limited processor speed, which means I can't make dt as small as I want. And with such big growth rates of the starfish, that can cause a serious difference in the solution of the model. But for this conclusion I will be ignoring these factors and base it off only the information the model gave to me.

We know that the Great Barrier Reef is a vulnerable ecosystem, and below I will give some mathematical reasons why.

One of the main issues lies in the eigenvalues. The point of equilibrium is always unstable. This would not be a problem if the solution started in the point of equilibrium, as it would stay there. But the initial values of S and C do not start inside the point. Even if with human help we managed to bring the values of S and C into the point of equilibrium, we would still not have saved the reef. This is because the point of equilibrium moves, since the model contains a cosine. If the point moves away, the solution is no longer in the equilibrium and will start to divert.

Typical predator-prey models have a high growth rate in the prey population, and the predators will need a large population of prey for their population to grow. The first characteristic is missing in my model of the Great Barrier Reef. The coral does not grow fast at all, and the starfish grow at a much larger speed. An outbreak of predators therefore consist of so many starfish eating coral that the damage done to the reef is enormous. If left uncontrolled, the starfish will eat almost everything, if not everything, of the coral on the reef. This in turn leaves the starfish population without food, and they will die.

Because the initial value of Coral allows the starfish to grow, we will have an outbreak of starfish in the future. In order to save both the coral and the starfish, we will need to prevent or limit the effects of this outbreak.

Then comes the question: How are we going to control this outbreak? Sadly, I do not have a precise answer. The most logical solution would be to systematically remove a certain amount of starfish from the reef on perhaps a monthly basis, but improving living conditions for predators of the starfish will also help. The fact that reducing the number of starfish recruited on the reef delays the date of the outbreak, as shown in figure 12 and 17, does suggest that this solution will be effective. Because the model is inaccurate about numbers, I cannot tell you the amount of starfish we need to remove from the reef.

Everything I have written above will only have an effect when the coral's growth rate is positive, and bleaching is infrequent. This means that it is also necessary to control the climate and sea temperatures.

To quickly summarize: my model does show how the parameters affecting the reef influence each other, but the numbers related to that are not yet believable. It might have been better to describe a section of the reef, rather than the entire reef, as the coral cover and starfish population do not have to be spread out evenly across the reef. This means I should have made several sub-models, all with different starting values. This way, I would have gotten more precise information of the reef, as in our current model the entire reef's populations change every timestep. Doing this does increase the amount of work to do, and since I already had the information I needed to answer my main question I decided to leave the model as is.

Furthermore, I would have to iterate on the growth rate of the starfish in order to get the numbers to be more realistic, which means I can actually test the results of certain actions people can take to try and secure the future of the reef.

Nonetheless, I think it is safe to say that the concerns about the reef are true, and that the future for the coral looks grim. Reducing the amount of starfish recruited on the reef does result in the reef surviving longer, which suggests that systematically killing starfish to prevent an outbreak would help saving the reef.

Activity Log

Date	Place	time	tasks performed	duration
Friday 22nd of February 2019	school	15:30- 14:00	brainstorming research questions	30 min
Wednesday 27th of February 2019	home	10:00 - 11:30	working on SRP proposals. Used https://en.wikipedia.org/wiki/Environmental_threats_to_the_Great_Barrier_Reef as inspiration.	90 min
Monday 11th of March 2019	school	10:20 - 11:00	Finalizing proposals and adding a source list. Tried to make a clear difference between the proposals.	40 min
Friday 31 May 2019	home	13:20 - 15:00	orientation on how to make a research plan. Writing of the research plan except for the background info and the planning purchasing agenda to help make the planning	100 min
Saturday 1 June	home	13:20- 15:10	Finalizing research plan: finishing planning and working on sub-question 1 and 2 finding sources: https://www.marineconservation.org.au/coral-bleaching/+ https://www.barrierreefaustralia.com/info/coral-facts/ https://link.springer.com/article/10.1007/BF00265013 https://www.int-res.com/404/ https://oceanservice.noaa.gov/education/kits/corals/coral04_reefs.html	110 min

			https://oceanservice.noaa.gov/education/kits/corals/coral06_reproduction.html http://www.marinespecies.org/ http://www.anemoon.org/ https://www.pnas.org/ https://www.aims.gov.au/-	
Tuesday 4 June	school	11:10 - 12:00	Finding sources and data required for the modeling sources found: https://pdfs.semanticscholar.org/e2bd/4de6724545094becd6d28de98ba76681d881.pdf https://www.forbes.com/sites/trevornace/2018/04/19/half-of-the-great-barrier-reef-coral-has-died-since-2016/ https://en.wikipedia.org/wiki/Great_Barrier_Reef https://www.aims.gov.au/documents/30301/2107187/cots-revised.pdf http://data.aims.gov.au/waCOTSPage/cotspage.jsp https://www.nature.com/news/2009/090323/full/news.2009.185.html?error=cookies_not_supported&code=642d6d80-86c5-40b9-b0fb-0c2170deaf97	50 min
Thursday 6 June	school	11:10 - 13:15	Finding sources: https://www.pnas.org/content/pnas/109/44/17995.full.pdf https://www.livescience.com/23612-great-barrier-reef-steep-decline.html	135 min

			Rewriting sub-question 1 to improve clarity	
June 7th	school	11:10 - 12:00 13:15 - 14:05	practiced using LaTeX2PNG and made model example for sub-question 1 latex commando learned: *\begin{bmatrix}* *X_1 & X_2 * *X_3 & X_4 * *\end{bmatrix}* written piece about Euler's method. Finalizing sub-question 1	100 min
June 14th	school	11:10 - 12:00	Working on sub-question 2: wrote introduction added illustrations to sub-question 1 and 2 sources used: http://www.greatbarrierreef.org/reef-experiences/	50 min
June	school	14:25 - 13:10	found source: https://en.wikipedia.org/wiki/Coral Looked up information and worked on sub-question 2 introductory paragraph	45 min
June 27th	school	9:00- 10:00 11:10 - 12:00	Worked on and finished sub-question 2 Found source https://en.wikipedia.org/wiki/Crown-of-thorns_starfish#cite_note-21 https://www.sciencedirect.com/science/article/pii/0022098192900186?via%3Dihub For the feeding rates of the starfish.	60min
July 21st	home	13:00 - 13:30 14:00 - 14:30	Worked on and finished introductory paragraph of sub-question 3. Worked on and finished modeling cycle 1 - problem and modeling phase.	60 min
July 22nd	home	12:45 - 13:45	Worked on and finished modeling cycle one- calculation and verification phase.	60 min

August 22nd	home	13:35-14:00	worked on and finished modeling cycle 2 problem and model phase	25 min
September 5th	school	10:20 - 12:00 12:25 - 13:10	finished cycle 2 calculation and verification phase. Found sources: https://www.reefresilience.org/pdf/COTS_Nov2003.pdf and http://data.aims.gov.au/waCOTSPage/cotspage.jsp Worked on and finished modeling cycle 3 problem phase, worked on cycle 3 model phase.	145 min
september 6th	school	9:00 - 11:50 20 minute break included	found source: https://www.researchgate.net/publication/237328832_Crown-of-thorns_starfish_and_coral_surveys_using_the_manta_tow_and_scuba_search_techniques#pf8 worked on cycle 3 modeling	150 minutes
September 18th	school	11:10 - 12:00 12:10-12;25 16:00-16:10	looked for documents about the growth rate of COTS. Found the following sources: (use control f to find the right part and search for %) http://elibrary.gbrmpa.gov.au/jspui/bitstream/11017/3257/1/Pratchett_etal_2017_Thirty_years_of_research_on_COTS.pdf https://www.mdpi.com/1424-2818/9/4/41/htm http://elibrary.gbrmpa.gov.au/jspui/bitstream/11017/3190/1/MORAN_ETAL_CROWN_OF_THORNS_STARFISH_SYNOPSIS_RESEARCH.pdf file:///home/chronos/u-0364ca8f68b0427ec3b315c7637ed55904a335a9/MyFiles/Downloads/BabcockandMundyAcanthasterReproAJMFR1992.pdf (This file is in my google chrome archive. Furthermore I discussed with Peters and Meerhof about whether these things would be logical and how I could implement them.	75 mins

September 20th	school	11:55-12:00	Checked whether the logbook was in the proper order together with Mrs. Kruithof	5 mins.
September 24th	school	14:20 - 15:10	finished modeling cycle 3 calculations and verifications phase. Edited Logbook based on Kruithof's comments. finished Cycle 4's problem phase.	50 mins
September 30th	school	11:10-12:00	worked on cycle 4 modeling phase used sources: http://elibrary.gbrmpa.gov.au/jspui/bitstream/11017/3257/1/Pratchett_etal_2017_Thirty_years_of_research_on_COTS.pdf http://elibrary.gbrmpa.gov.au/jspui/bitstream/11017/3190/1/MORAN_ETAL_CROWN_OF_THORNS_STARFISH_SYNOPSIS_RESEARCH.pdf	50 mins
October 4th	School	11:10 - 12:00	Discussed with Mrs. Molenaar about cycle 4. Finished the model for Cycle 4.	50 mins.
October 7th	School	11:10-12:00	Worked on cycle 4 modeling phase, which simply included typing out what I created last friday.	50 mins
October 8th	School	9:30 - 10:00 14:20 - 15:10	Worked on cycle 4 modeling phase: typing out my line of thought from last friday	80 mins
October 10th	Home	16:45 - 17:20	Ran the model in my computer with Python	45 minutes
October 11th	School	11:10 - 12:00	Worked on calculations phase and verification phase of cycle 4, made model for cycle 5	50 minutes
October 14th	School	11:10-12:00	Worked on and finished cycle 5 modeling phase	50 minutes
October 16th	School/home	8:30-9:10 10:25-12:00 12:00 - 15:10	Tried to solve the issue of my computer not being able to calculate the model (dt too small, but larger is impossible)	325 min

October 22nd	Home	13:00-13:30	Wrote mail to Dennis den Ouden	30 mins
October 30th	School	12:40 - 13:10 14:30-15:10	worked on making final changes to the model which were suggested by Dennis. Worked on solving model to find the points of equilibrium. Found points of equilibrium but encountered a problem with the fact that S is dependent on itself. thought about way to circumvent/approach the equilibrium value.	70 mins.
November 1st	School	11:15-12:00	Worked on introductory section	45mins
November 15th	School	9:20-9:45	solved the issue with S_eq being dependent on itself, as described on October 30th. worked on finding final functions/points of equilibrium.	25 minutes
November 18th	School	12:45-13:00	checked whether F would be nearly 1 in the point of equilibrium (related to the issue of oct. 30th). Found a mistake in point of equilibrium value and corrected it.	15 min
November 19th	School	13:10 - 14:00	Found definitive points of equilibrium. Worked on digitalising my calculations with LaTeX (worked on modeling cycle 5 calculations phase).	50 min
November 20th	School	11:10 - 13:30 (breaks excluded)	Worked on finding the eigenvalues	120 min
November 21st	School	11:10-12:00	Worked on finding the eigenvalues	50 minutes
November 22nd	School	11:10-12:00	Wrote my calculations in the working document with the help of LaTeX. Finished digitalising the calculations for the point of equilibrium.	50 min
November 27th	School	11:10-12:00	worked on digitalising calculations with LaTeX2PNG	50 min
Nov. 28th	School	11:10-12:00	Worked on digitalising calculations and found LaTeX2PNG symbol for \partial (for	50 min

			partial differentiated functions)	
Nov. 29th	School	11:10-12:00	Worked on digitalising calculations for the partial differentiated functions	50 mins.
Dec. 3rd	School	14:20-15:10	Finished calculating the eigenvalues for the summer	50 mins.
Dec 5th	School	11:11 -11:50	Finished calculating the eigenvalues for winter. Finished calculations phase.	40 mins
December 9th	School	11:10-12:00	Finished verification phase of cycle 5. Worked on organizing source reference list.	50 mins
December 11th	School	11:10-12:00	Worked on introduction, conclusion, and front page.	50 mins
December 28th	Home	11:20 - 12:10 14:30-16:00	Made annotations in the text, checked if the links were working and re-annotated certain sources when possible if they didn't. Made phase diagram for cycle 5 with the point of equilibrium plotted.	140 mins
December 31st	Home	10:30 - 11:00	Resized latex equations, added figure numbers. Found source that also includes the 50mil eggs from COTS (to correct that link 2 is not working) https://www.cell.com/current-biology/pdf/S0960-9822(13)00969-X.pdf	30 min.
January 5th	Home	11:45-12:30	Fine-tuning, checking paper with the help of the assessment form, running experiment with reduced egg-laying of S. Made list of assumptions.	45 min
January 7th	Home	15:00-16:00	Fine-tuning, checked spelling	60 min
January 28th	School	14:20-14:45	Made changes according to comments of Molenaar	25 min
January 29th	School	11:10-12:00	Worked on powerpoint	50 min
February 3rd	School	11:10-12:00	Worked on Powerpoint, gave permission to Molenaar to send my PWS to Pythagoras	50 min

February 9th	School	12:30-13:00 13:30-14:30	Improved diagrams in document as suggested by Mrs. Molenaar. Turned sub-questions into sub-sections.	90 mins
February 12th	School	11:40-12:00	Wrote summary, added new assumption of assuming climate change = nonexistent (climate change influences growth rate of coral)	20 mins
February 16th	School	11:10-11:30	Spelling-check and other final touches. Handed in digital paper, as well as a physical black/white copy	20 mins

Sources

These are all the websites I have visited during my research. I may have used them for information, but also for inspiration.

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12. Kaplan, M. (2009, March 24). Coral may live for thousands of years : Nature News. Retrieved June 4, 2019, from https://www.nature.com/news/2009/090323/full/news.2009.185.html?error=cookies_not_supported&code=642d6d80-86c5-40b9-b0fb-0c2170deaf97

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27. http://www.anemoon.org/

28. https://www.pnas.org/

29. https://www.aims.gov.au/

Appendix

Python code for cycle 5:

```
# -*- coding: utf-8 -*-
"""
Created on Wed Oct 16 08:36:02 2019

@author: Floris Peter Meijvis
"""
# Program      : Euler's method for a system
# Author       : MOOC team Mathematical Modelling Basics
# Created      : May, 2017
import numpy as np
import matplotlib.pyplot as plt
import math

# Initializations
Dt = 0.0000002                        # timestep Delta t
C_init = 77499000000                  # initial population of Coral
S_init = 64582502                     # initial population of Starfish
t_init = 0                            # initial time
t_end = 2                             # stopping time

n_steps = int(round((t_end-t_init)/Dt)) # total number of timesteps

X = np.zeros(2)                      # create space for current X=[P,G]^T
dXdt = np.zeros(2)                   # create space for current derivative
t_arr = np.zeros(n_steps + 1)        # create a storage array for t
X_arr = np.zeros((2,n_steps+1))      # create a storage array for
X=[P,G]^T
t_arr[0] = t_init                    # add the initial t to the storage
array
X_arr[0,0] = C_init                  # add the initial P to the storage
array
X_arr[1,0] = S_init                  # add the initial G to the storage
array

# Euler's method
for i in range (1, n_steps + 1):
       t = t_arr[i-1]                    # load the time
       C = X_arr[0,i-1]                  # load the value of P
       S = X_arr[1,i-1]                  # load the value of G
       X[0] = C                          # fill current state vector
X=[P,G]^T
       X[1] = S
       if S > 0:
```

```
            dCdt =
(0.0089*(1-(C/344440000000))*C-(0.0161+abs(0.02565*math.cos(math.pi*t))
)*S) *(1/100000)
# calculate the derivative dP/dt

            dSdt =
(((C/34444000000)-1)*2500000*(0.5*2**(math.sqrt(344440000000/S)/-30)+0.
5*2**(2*math.sqrt(344440000000/S)/-30))*S - (1/16)*S)*(1/100000)
.                                          # calculate the derivative
dG/dt
            dXdt[0] = dCdt                 # fill derivative vector
dX/dt
            dXdt[1] = dSdt
            Xnew = X + Dt*dXdt             # calculate X on next time
step
            X_arr[:,i] = Xnew              # store Xnew
            t_arr[i] = t + Dt              # store new t-value
        else:
            dCdt =
(0.0089*(1-(C/344440000000))*C-(0.0161+abs(0.02565*math.cos(math.pi*t))
)*S)*(1/100000)                                # calculate the
derivative dP/dt
            dSdt = 0                       # calculate the derivative
dG/dt
            dXdt[0] = dCdt                 # fill derivative vector
dX/dt
            dXdt[1] = dSdt
            Xnew = X + Dt*dXdt             # calculate X on next time
step
            X_arr[:,i] = Xnew              # store Xnew
            t_arr[i] = t + Dt              # store new t-value

# Plot the results
fig = plt.figure()
plt.plot(t_arr, X_arr[0,:], linewidth = 4, label="C(t)")  # plot P vs.
time  (to make it a phase-diagram remove t_arr and replace it with
X_arr[1,:])
plt.plot(t_arr, X_arr[1,:], linewidth = 4, label="S(t)")  # plot G vs.
time

plt.title('Coral and Starfish', fontsize = 20)     # set title
plt.xlabel('t (years)', fontsize = 20)             # name of horizontal
axis
plt.ylabel('C(t) in m^2 and S(t) (dimensionless)', fontsize = 20) #
name of vertical axis

plt.xticks(fontsize = 15)                 # adjust the fontsize
plt.yticks(fontsize = 15)                 # adjust the fontsize
```

```python
plt.axis([0, 2, 0, 350000000000])        # set the range of the axes

plt.legend(fontsize=15)                   # show the legend
plt.show()                                # necessary for some platforms

 # save the figure as .jpg (other formats: png, pdf, svg, (ps, eps))
fig.savefig('PWS 5.jpg', dpi=fig.dpi, bbox_inches = "tight")
```